Nuclear reactor safety:
Multiplicity Distribution of Prompt Neutron Fission, Frehaut and Terrel Comparison
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ABSTRACT : multiplicity of neutrons when a fission occur has probability laws given by two authors, Terrel and Frehaut**.**

This intrinsic random phenomena can have important implications on nuclear safety ("when a reactor start**,** there is, then, for example, a certain probability that the reactor may go beyond prompt critical before any neutron signal is detected" **-** Bell & Glasstone, Nuclear Reactor Theory, ed Van Nostrand, 1970,Reinhold, page 36). Its study is so important. I show this study must be completed.

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I) <u>Introduction : fluctuations and Boltzmann equation</u>

Man use currently chemical energy since many years, 500 000 for fire, without any bigs problems. Even if sometimes this dragon escape from its hands, the damages aren't too excessive.

Nuclear energy is only in use since an instant, opposite to the fire. But its power is more strong, many order of scale. So, an accident can injure millions peoples. If we want to use safety for thousands years this splendid energy, it is necessary to study it very carefully in all its aspects, without taboos.

The simulation of start of a nuclear engine turns aside from the beaten track. I want to say there is not only the classical Boltzmann equation. Of course it's a very important equation, but it must not mask underlying phenomenas, radio active behaviour of fission, which are of greatest important, particularly in this phase.

Indeed, the radio actives phenomenas have intrinsics uncertains (for don't say random) behaviours. This behaviour is taking into account by probabilities laws. The determinist Boltzmann equation use, in classical neutronic (in classical statistic mechanic too), cross sections as a traduction of this « random » behaviour, and it gives <u>only the mean value of the number of neutrons</u>. But there is another spring of uncertainty we don't see with this equation. It's the probability (US says multiplicities) that a fission give 0, 1, 2 …, 7, etc… neutrons. **This involve intrinsics fluctuations of the number of neutrons in an engine**, Theses fluctuations are not dues to imperfects technologies, but to radio actives naturals process that man cannot master. **Can theses fluctuations be dangerous ?**

Of course this phenomena is known since many times, and a pioneer of its study was the Nobel price Feynman [*1*].

However quantification of these probabilities emission of neutrons by fission was made at the end of the fifties [2], then in 1988 by J. Frehaut of *CEA/Division des Applications Militaires* [3] who offer a probability law of events with an anomaly : sum of probability of all events not to equal to one.

The more deep equation managing the neutronic is the backwards Kolmogorov equations. Theses equations was established first by Hungarian engineer L. PàL [*4*].

Pàl, in his pioneering work, use a point reactor model. It is George Bell, of Los Alamos, who take more clearly, in 1963, account of space and energy dependence of neutrons [5].

Fundamentals data for use theses works are the probability of number of neutrons emitted by fission.

I will present comparison of Terrell and Frehaut distribution law, with some consequences.

« **There is then, for example, a certain probability that the reactor may go beyond prompt critical before any neutron signal is detected** » ; Bell & Glasstone (Nuclear Reactor Theory, ed Van Nostrand, 1970,Reinhold, page 36)

II) The mathematical formulation of the distributions of the number of neutrons emitted by fission

I will not comment how, and what hypothesis was taken by the authors to establish their distributions laws.

I will only present their formulation. I note P_ν the probability to have ν neutrons. P_ν depend of the energy of the incident neutron which fission the target nucleus. This dependence is implicitly expressed by the mean number of prompt neutrons per fission, $\bar{\nu}_p$.

a) Terrell distribution

P_ν is defined with a cumulative formulation : $\sum_{\nu=0}^{n} P_\nu = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(n-\bar{\nu}_p+1/2)/\sigma} e^{-t^2/2} dt$.

Compute P_ν is easy knowing that $\int_{-\infty}^{a} e^{-\frac{x^2}{2}} dx = \frac{\sqrt{2\pi}}{2}\left[1+erf\left(\frac{a}{\sqrt{2}}\right)\right]$. Terrel use σ =1.08 for Pu^{242}, Pu^{240}, Pu^{238}, U^{235}, U^{233}, Cm^{242}, Cm^{244} and use σ = 1.21 for Cf^{252}.

This model is used by Mac Cullen in his code, TART [6] (but he doesn't say if he uses σ = 1,21 forCf^{252}, so this must be verify - no mention of thorium in his article -)

b) Frehaut distribution

We have the very simple expression $P_\nu = \frac{K}{\sigma\sqrt{2\pi}} e^{-\frac{\left(\frac{\nu-\nu_p}{\sigma}\right)^2}{2}}$, where K and σ are give by

ν	0	1	2	3	4	5	6	>7
σ	0,94	1,13	1,22	1,295	1,16	1,222	1,226	1,235
K	2,827	1,073	1,075	1,095	0,953	0,958	1,048	1,000

Frehaut say that his data are valid for all uranium and plutonium isotopes, but not for thorium isotopes.

c) Nota :

c.1) As for Frehaut, Terrell **data lack too for thorium isotopes**, which can become very important as energy source in future.

c.2) A report of IAEA give data for Pu^{239} at 80 KeV and for $\bar{\nu}_p$ = 3,035

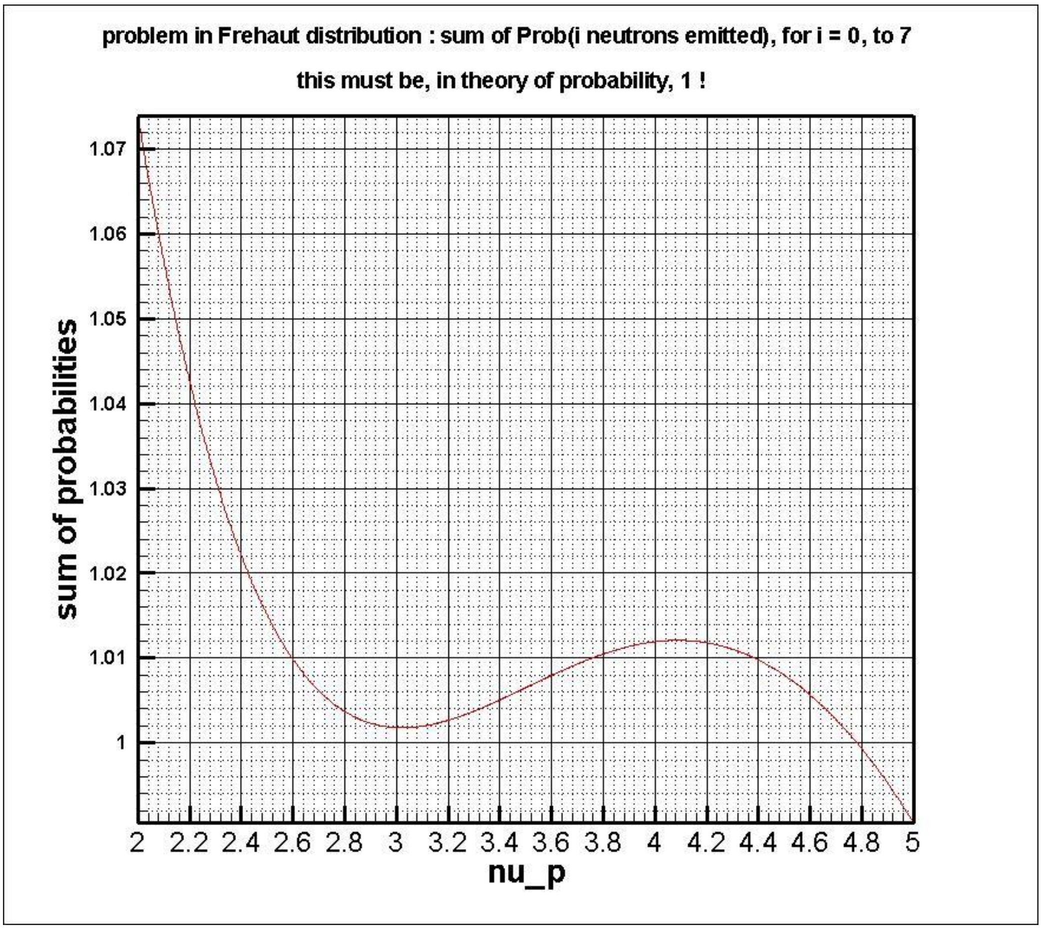
P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7
0,001	0,106	0,125	0,538	0,106	0,058	0,048	0,009

Frehaut distribution give

ν	P(ν)
0	6.535783589485982E-003
1	7.483834377840093E-002
2	0.245272595011948
3	0.337205800632433
4	0.231896389765082
5	8.585564606391487E-002
6	1.831458755025303E-002
>7	1.866797809085332E-003

We can see these data are incoherents

c.3) surprisingly, Frehaut data doesn't respect the first rule of probability law : the set of all the events has probability non equal to 1 !

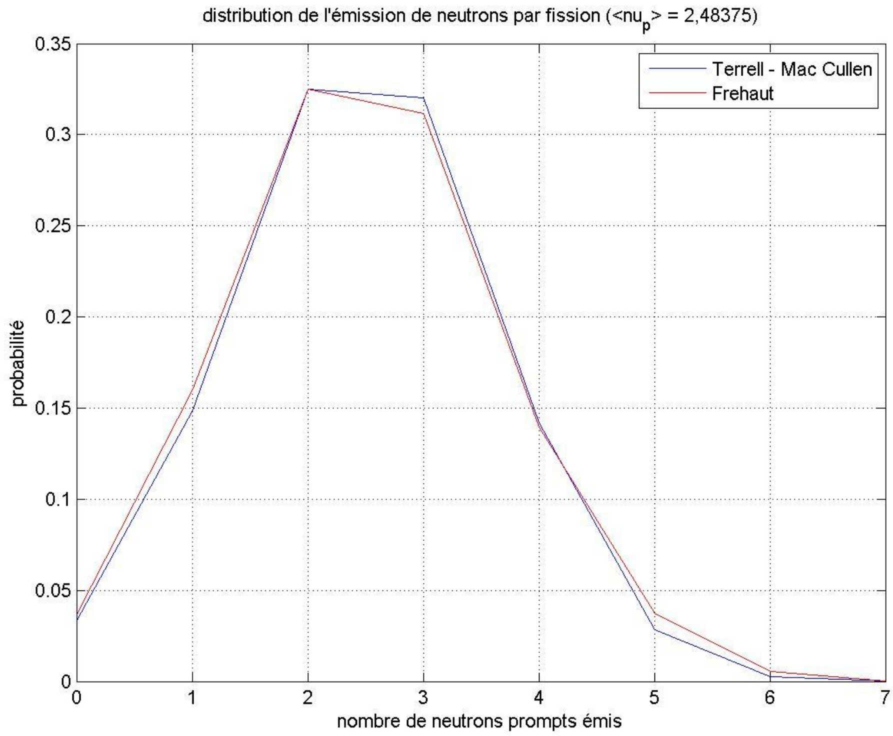


An US author note this lack of consistency (*Steven Douglas Nolen, thesis Los Alamos 13721, year 2000*) and make a normalisation « to have the least impact on the most likely portion of the Frehaut distribution's shape » !

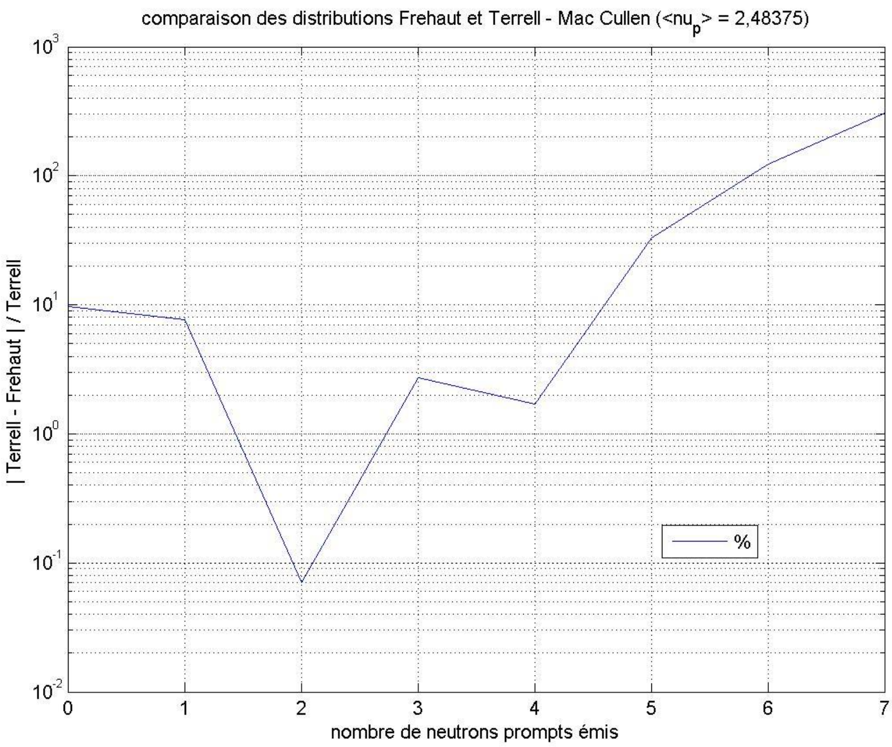
III) Simple comparison between the distribution of Frehaut and Terrel

The same author note effect of neutron multiplicitiy on chain length distribution (page 67) but compare binary distribution with Frehaut distribution, and doesn’t use Terrell distribution.

First we compare purely and simply the data for rapids neutrons



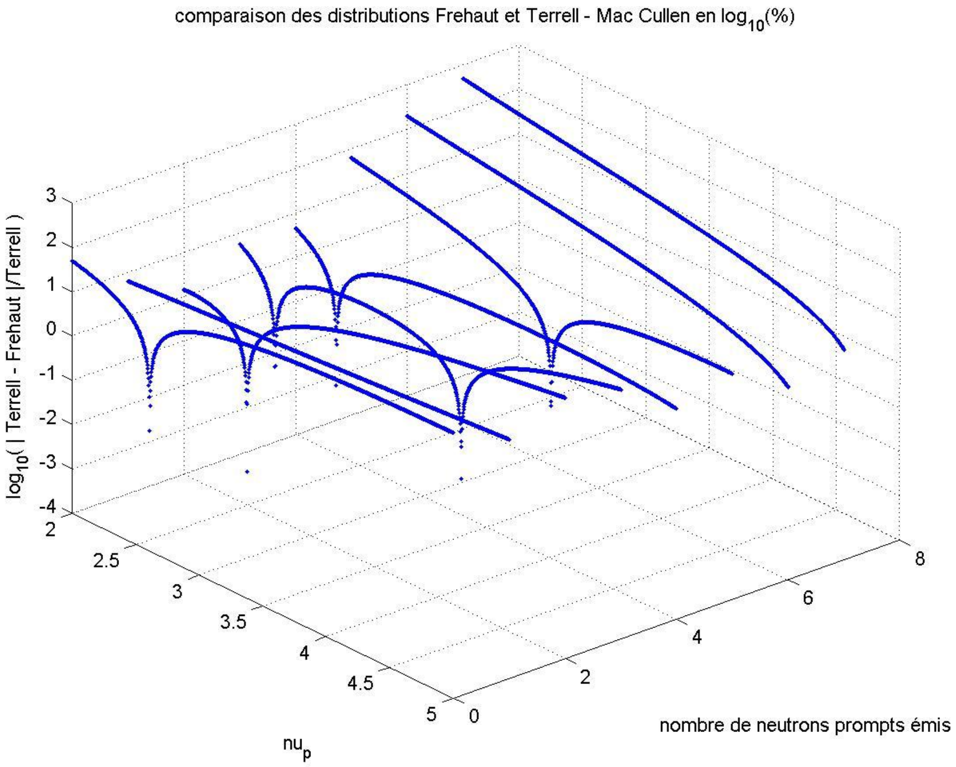
We can see the difference is pronounced for 3 emitted neutrons. The relative difference is plotted in % (Terrel is 100 %) :

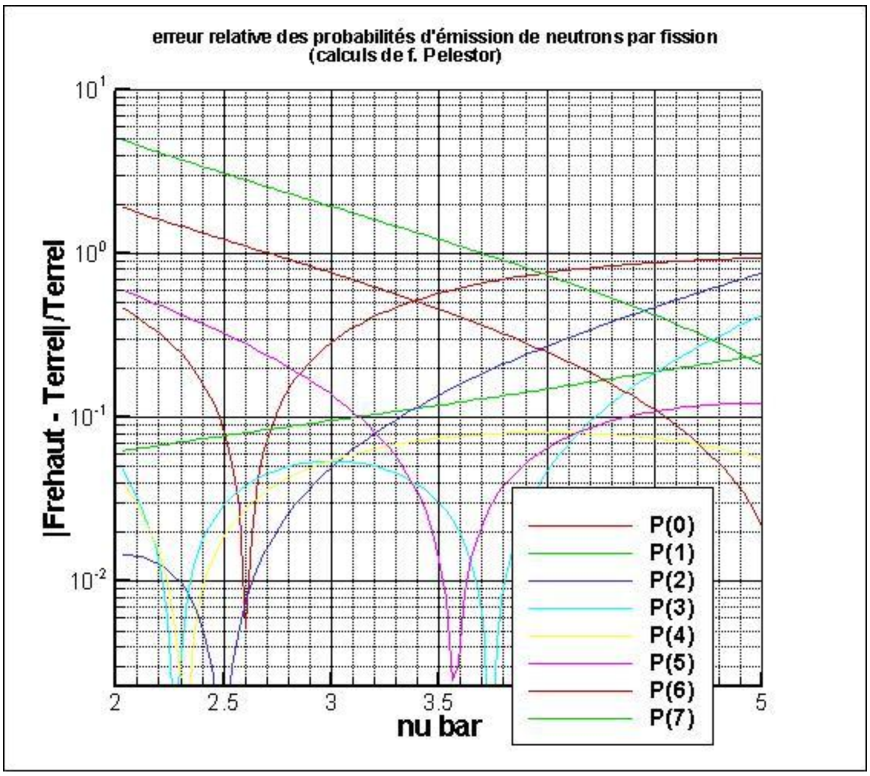


We see that the minimum is for number of prompt neutrons emitted equal 2, for this energy.

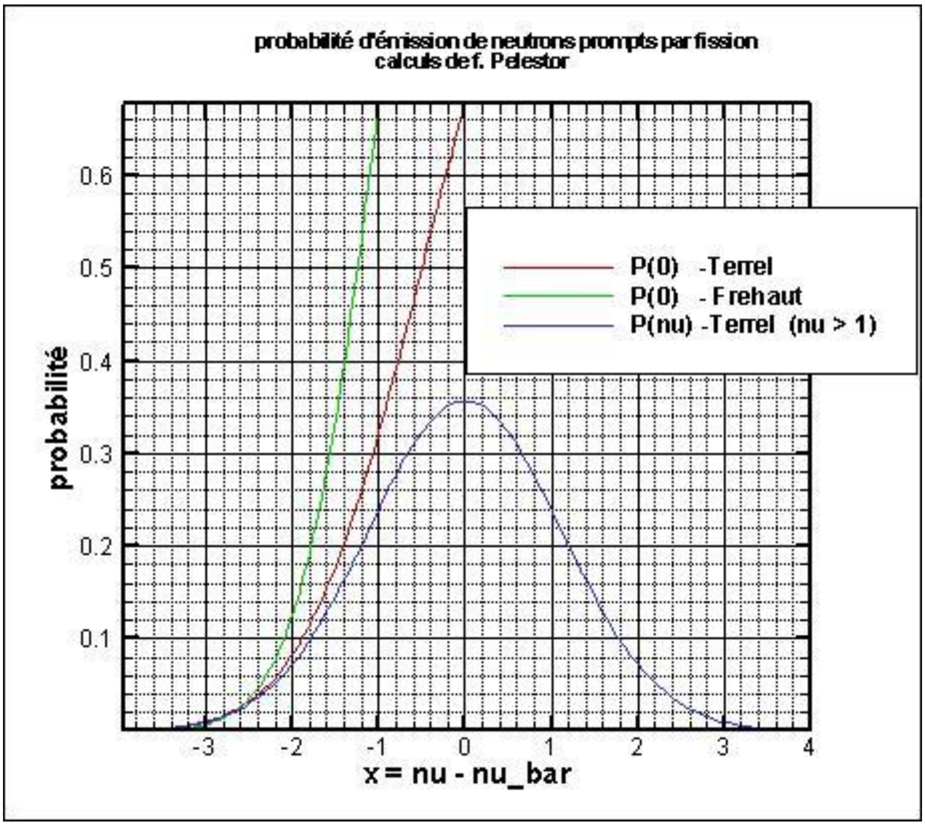
In fact it is interested to see what happen for different incident neutron energy, which correspond to interval of variation of $\bar{\nu}_p$, here in [2, 5]. In 3D figure the difference is in % (100 ù = Terrell), in 2D figure it is not.

For example, 14 Mev thermonuclears incidents neutrons corresponds near to 4 for $\bar{\nu}_p$ (janis)





It is interested to compare with $\nu - \bar{\nu}_p$, where ν is the number of prompt neutron emitted :



I have no put Frehaut probability for $\nu \geq 1$. We can see the difference for 0 neutrons emitted.

IV) <u>Consequence on neutrons number distribution in a nuclear engine</u>

The evolution of the neuton population in an engine can be defined by the probability to raise a level, $P(n,\vec{m},t)$, which is the probability to have n prompts neutrons, $\vec{m} = (m_1, m_2, m_3, m_4, m_5, m_6)$ delayed neutrons (in fact precursors) of type 1, 2, …,6 at time t.

This representation allow to take account of the fluctuations which can arrive in all the phase (start, working speed, stop), particuliarly during start and stop.

I will consider a point model reactor (without enrgy and space dependence) for clearness. All the information on neutrons population is contents in theses $P(n,\vec{m},t)$. We are interested by compute them to derive some interesting informations, as the **probability to initiate a chain reaction, the pic power we can have during fluctuations**, etc …

A method to have these numbesr is to solve a partial differential equation where unknown is the function $g(x,\vec{y},t) = \sum_{n=0}^{\infty} \sum_{m_1=0}^{\infty} \ldots \sum_{m_6=0}^{\infty} x^n y_1^{m_1} \ldots y_6^{m_6}\, p(n,\vec{m},t)$ with $\vec{y} = (y_1,\ldots,y_6)$. The backward Kolmogorov equations, which describe the history of a neutron in the engine, give this equation :

$$\boxed{\frac{\partial g}{\partial t} = g\left[\sum_{o=1}^{s}(x^o - 1)S_o(t)\right] + \frac{\partial g}{\partial x}\left[\frac{-x}{l} + \frac{p_{cs}(t)}{l} + \sum_{i=0}^{6}\sum_{\nu=0}^{8} x^\nu y_i \frac{p_f(t)P_{\nu,i}}{l}\right] + \sum_{i=1}^{6}\frac{(x-y_i)}{\tau_i}\frac{\partial g}{\partial y_i}}$$

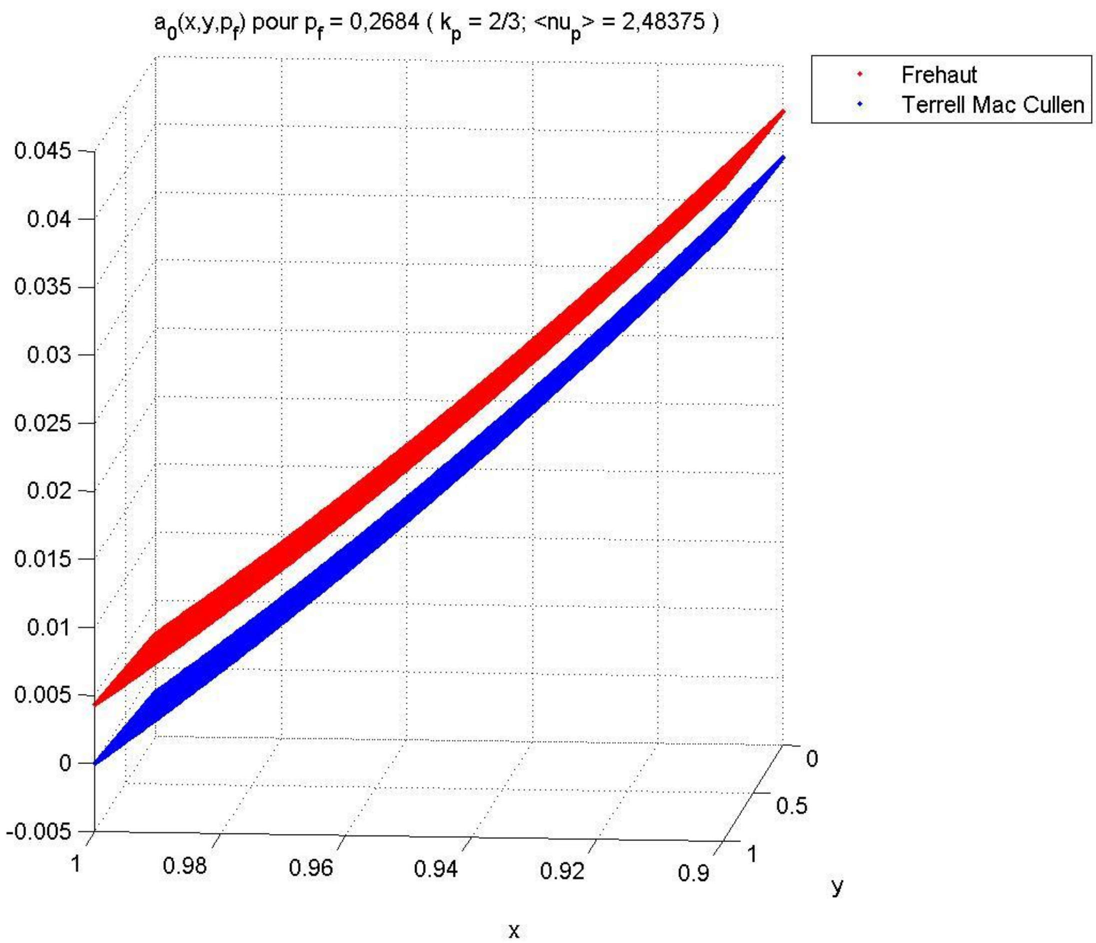
Here, $S_o(t)$ is a source which emits k neutrons during interval dt, starting at t. The term $p_{cs}(t)$ represent the probability of capture and leakage of the neutrons, $p_f(t)$ the fission probability ($p_f + p_c = 1$), l is the mean neutron lifetime, τ_i is the mean lifetime of the precursor of type i.

Not all of theses quantities are concerned by this discussion, but the $P_{\nu,i}$. This is the probability to have ν neutrons prompts when a fission occur (just what we have modelised with the two distributions of Terrell or Frehaut), but with a delayed neutron of type i (among 6, or 8 for the last evaluation). Of course we have $P_\nu = \sum_{i=0}^{6} P_{\nu,i}$. So the data about P_ν are importants.

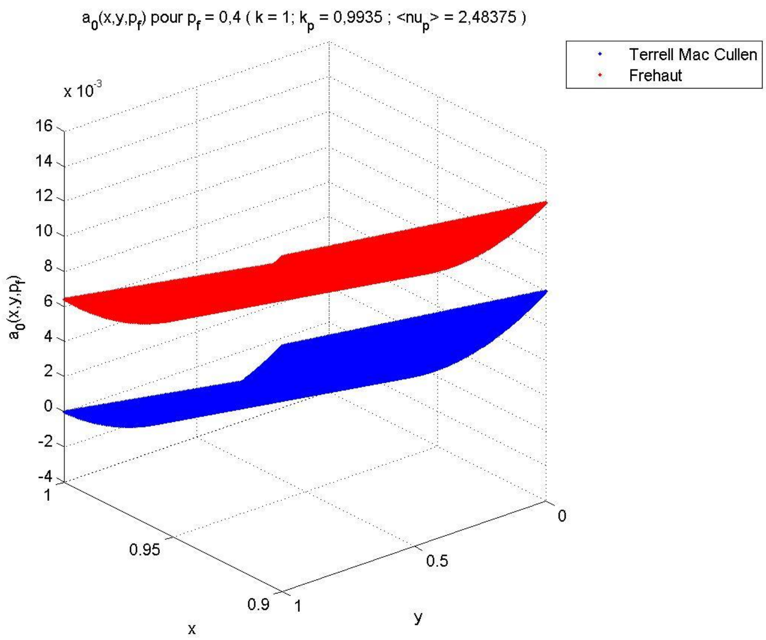
I will not discuss here about the interesting means to solve this equation, but we will look about the coefficients of $\frac{\partial g}{\partial x}$ which i name $-\frac{1}{l}a_0(x,\vec{y},t)$. I will present view of $a_0(x,\vec{y},t)$ for differents probabilities of fission p_f , which is related to $\bar{\nu}_p$ by the relation $p_f(t)\bar{\nu}_p = k_p(t)$, where k_p is the prompt criticity value. I will distinguish between subcritical and supercritical engine (with delayed neutrons).

Before, i must say that we are interested in real plan near x = 1 to compute P(n,t). It is why i show the a_0 coefficient in a near region of x = 1.

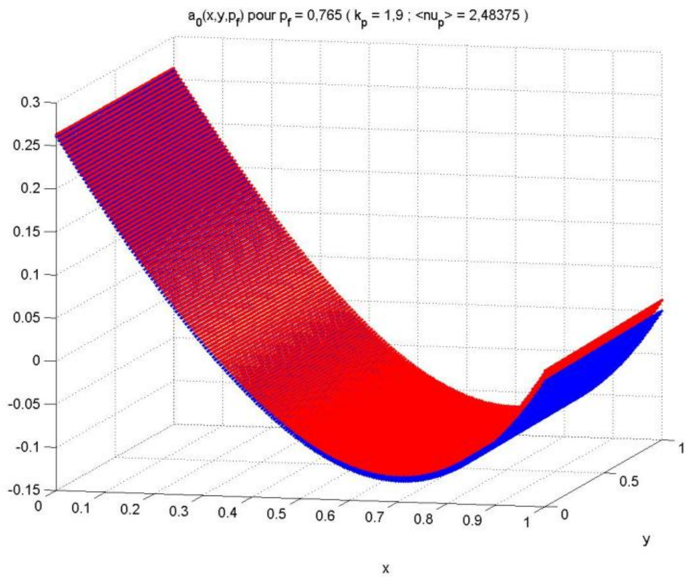
We can see, for subcritical engine that the Terrell distribution has negatives values for some position of (x,y), and that Frehaut has always positive values. This has consequence on caracteristics evolution of the partial differential equation of g.

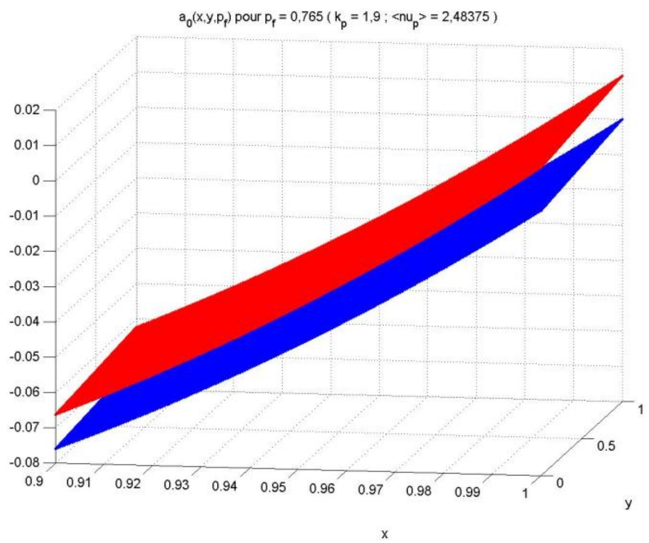


For near prompt critical engine, we have, for example with k_p = 0,9935 , the same behaviour :



and for a high supercritical engine (in red Frehaut and in blue Terrell)

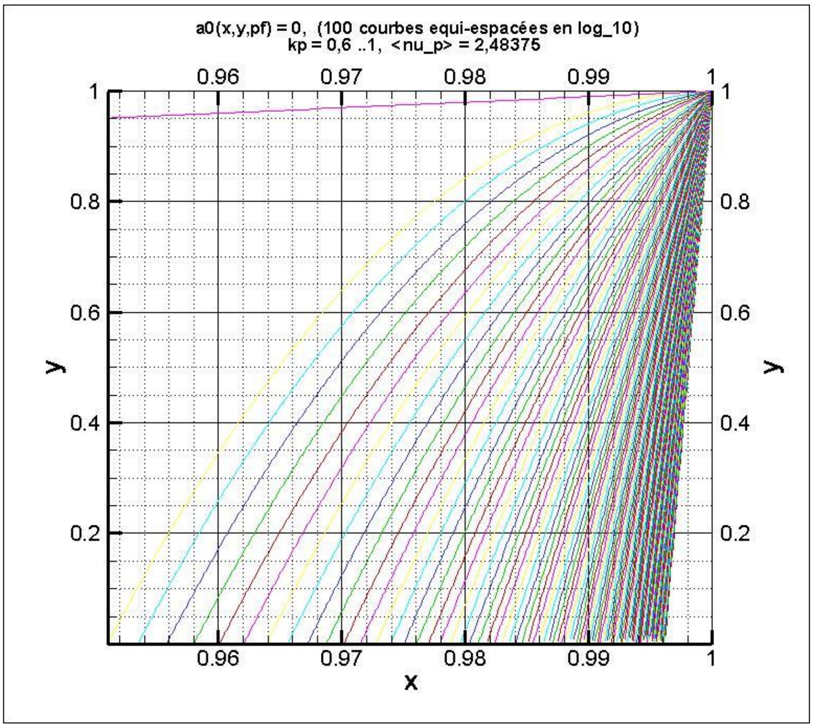




The difference is more pronounced than in subcritical engine, but, here, all the two have negatives values.

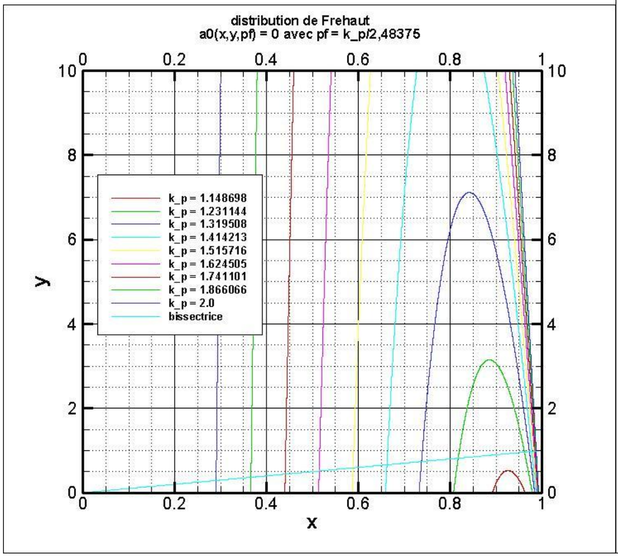
So, the computing of the function g will give differents values if we use Terrell or Frehaut distribution.

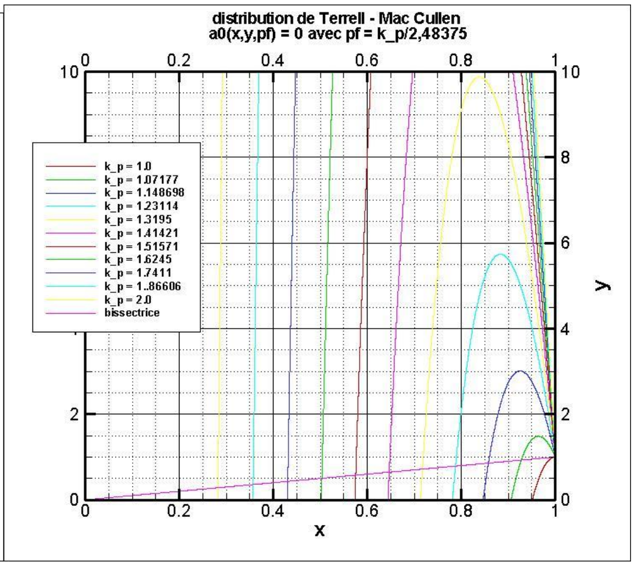
On this figure we can see the curves y(x) so than $a_0(x,y(x),p_f) = 0$ for the Terrel distribution and subcritical engine :



Theses curves simply doesn't exist with Frehaut distribution ! The high purple straight is the bisector of the plan (x,y).

For supercritical engine, this is different. The two distributions have curves which cut bisector, but Frehaut less, and at differents intersections positions.





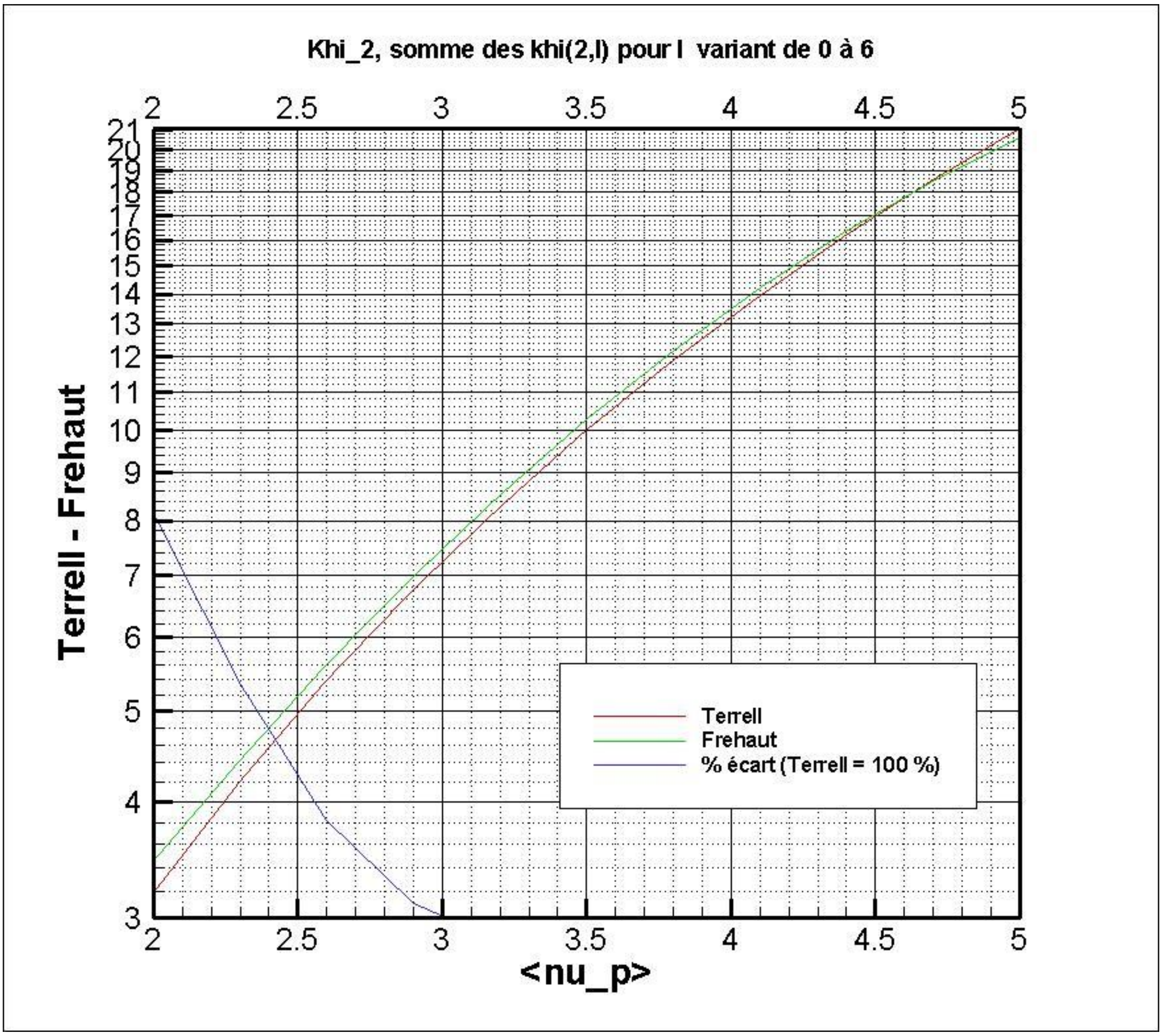
V) <u>The coefficient χ_2 and the probability to initiate a chain reaction</u>

The study of the probabilities $P(n,\vec{m},t)$, and the initiation of a chain reaction can be solved more simply with **some physicals approximations** of the coefficient $a_0(x,y,t)$.

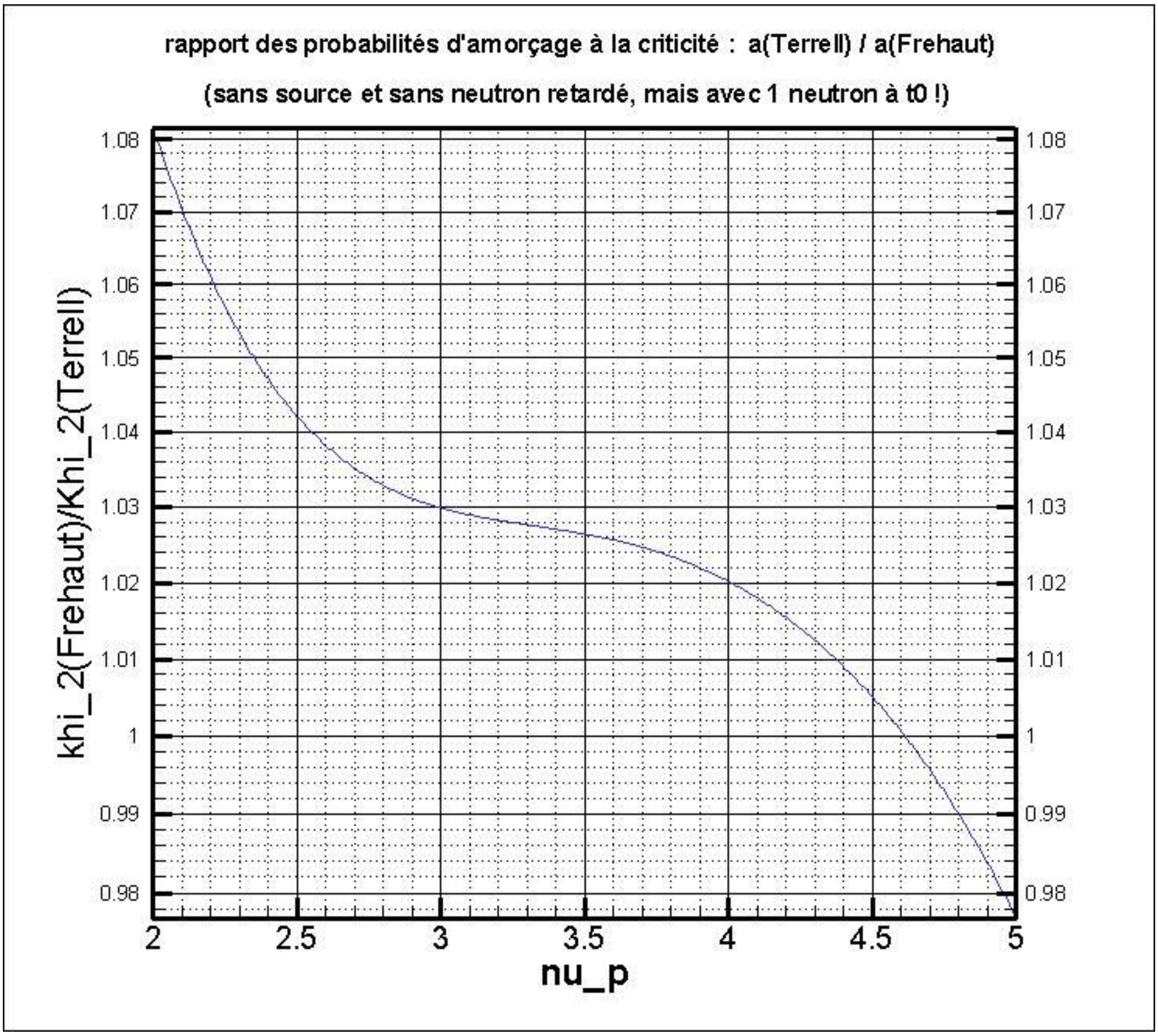
This lead to the $\chi_{k,l}$ coefficients, and particularly the χ_2 which is $\chi_2 = \sum_{l=0}^{6} \chi_{2,l}$.

Note that $\chi_{k,l} = \sum_{i=0}^{7} k!C_i^k P_{i,l}$ is the number of k-uplets prompts neutrons formed by fission with precursors of type l. χ_2 is just the number of pairs of prompts neutrons emitted.

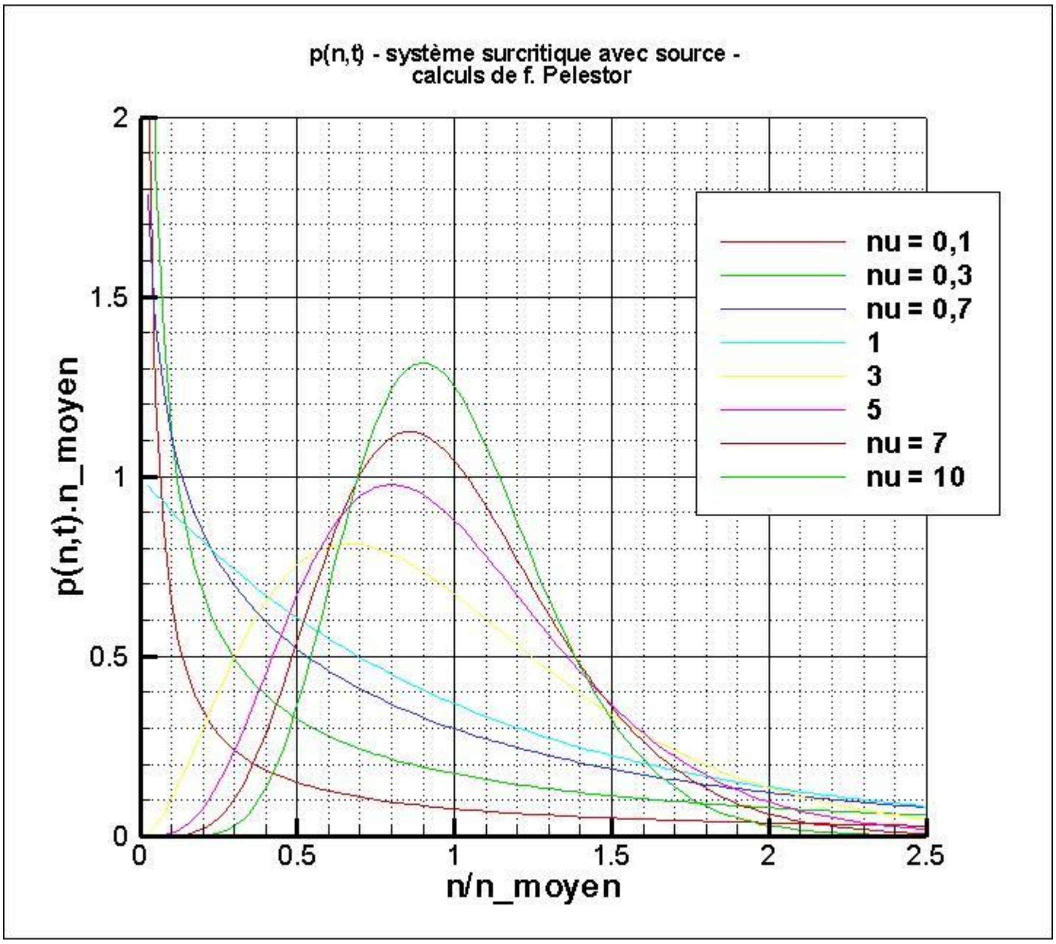
We can see the difference between the two distributions of this important quantity



For finish i will present the influence of theses distributions on the **probability of initiate a chain reaction** when there is no source but an initial neutron at time $t_{0,}$ when the engine become just critical, and when α is function of time. This was obtained by solving the partial differential equation without delayed neutron (however, here, value of χ_2 is obtain with delayed neutron) :



For surcritical engine with source ($g(x,\vec{y},t)$ is computing without delayed neutrons) , we have only to use the relation $\frac{nu_{Terrell}}{nu_{Frehaut}} = \frac{\chi_{Frehaut}}{\chi_{Terrell}}$ with the figure



This figure present probability of initiation of a chain reaction, P(n,t), multiply by mean value $\bar{n}(t)$ (« n_moyen » here) of n(t), versus $\frac{n}{\bar{n}(t)}$. The legend shows the trend for differents values of nu.

Note that the surface under every curves, $\int_{n_1/n_{moyen}}^{n_2/n_{moyen}} P(n)dn$, between the verticals abscissas n_1/n_moyen and n_2/n_moyen give the probability to have between n_1 and n_2 neutrons in the engine, when the mean is n_moyen.

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Ad Honores

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